Encapsulation theory: the minimised, uniformly violational radial branch.

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Abstract

This paper investigates how the hidden elements of a single radial branch should be distributed so as to minimise the branch's potential coupling.

Keywords

Encapsulation theory, radial encapsulation, branch, potential coupling.

1. Introduction

In the absolute encapsulation context when an encapsulated set is uniformly-distributed in violational elements, then its potential coupling is minimised when it is also uniformly-distributed in hidden elements (see proposition 1.11 in [1]). In the radial encapsulation context, however, this is not the case. This paper demonstrates this phenomenon using an example branch and derives the equation showing the hidden element distribution that achieves a minimum potential coupling.

This paper considers sets of radial information-hiding only.

2. Selected branch potential couplings

Consider the branch shown in figure 1, showing a branch of three disjoint primary sets, where the information-hiding and the information-hiding violation of the sets are shown within their representative symbols.

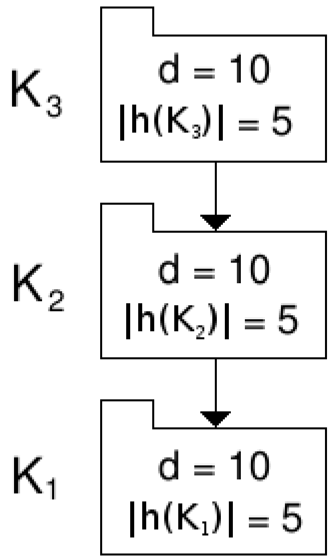

Figure 1: A branch of three disjoint primary sets.

Thus, taking the root set, K_1, for example, we see that K_1 has 10 violational elements, $|v(K_1)|$ *=10*, and 5 hidden elements, $|h(K_1)|$ *=5*; as we are concerned here with uniform violational distributions, we will discuss the specific violation of each set, and use the variable that we normally use for specific violations, hence *d=10*, though the meaning of both is, of course, the same. We note that violational and hidden elements of the branch are distributed uniformly throughout the sets.

Let us calculate the potential coupling of this branch.

The potential coupling of K_3 is the sum of its internal and external potential couplings. The internal potential coupling of K_3 is the total number of contained elements multiplied by this number minus one, i.e., *15x14=210*. The external potential coupling of K_3 is the total number of contained elements multiplied by the total number of violational elements it can see "below" it, i.e., *15x(10+10)=300*. So the potential coupling of K_3 is *210+300=510*.

As K_2 has the same number of contained elements then it will have the same internal potential coupling as K_3, i.e., *210*. K_2 can only see *10* violational elements "below" it, so its external potential coupling is *15x10=150*. So the potential coupling of K_2 is *210+150=360*.

K_1 has the same internal potential coupling as the other two, i.e., *210*, and K_1 has, like all root sets, no external potential coupling (there are no sets "below" it for it to see), so its potential coupling is the same as its internal potential coupling, *210*.

The branch's potential coupling is then: 5*10+360+210=1080*.

We know that, in the absolute encapsulation context, potential coupling is minimised (ignoring A.M.C.s) when both the violational and hidden element are distributed uniformly over all sets. Let us test to see whether this also holds in the radial encapsulation context: let us move a hidden element from K_2 to K_1, see figure 2.

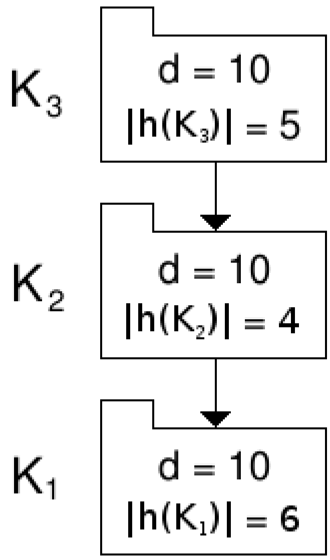

Figure 2: A branch of three disjoint primary sets.

Now let us calculate the potential coupling of this modified branch.

The potential coupling of K_3 is unchanged.

The potential coupling of K_2 has now fallen from *360* to *14x13+14x10=322*.

The potential coupling of K_1 has now risen from *210* to *16x15 = 240*.

The branch's total potential coupling has now fallen from *1080* to *510+322+240=1072*.

Thus by moving a hidden element, thereby rendering the branch non-uniformly distributed in hidden elements, we have reduced the branch's potential coupling. We note that the branch is still uniformly distributed in violational elements. This then raises the obvious question: given a branch that is uniformly distributed in violational elements, what distribution of hidden elements will minimise its potential coupling?

This question is of practical relevance as we suspect that a branch's minimum potential coupling will be achieved when each set in the branch has the minimum number of violational elements, and that minimum is one. In putting one element in each set, however, we are automatically uniformly distributing the violational elements, hence the configuration of a branch at its minimum potential coupling is one in which it is uniformly distributed in violational elements.

It can be shown that, to attain the minimum potential coupling in this case, the number of hidden elements in the i^{th} disjoint primary set is given by the following equation (see proposition 9.12):

$$|h(K_i)| = \frac{n}{r} + \frac{(r-3-2i)d}{4}$$

With this equation we can calculate the number of hidden elements in each of the three sets examined above to minimise the potential coupling.

K_3 should contain the following number of hidden elements:

$$|h(K_3)| = \frac{45}{3} + \frac{(3-3-6)10}{4} = 0$$

K_2 should contain the following number of hidden elements:

$$|h(K_2)|=\frac{45}{3}+\frac{(3-3-4)10}{4}=5$$

K_1 should contain the following number of hidden elements:

$$|h(K_1)|=\frac{45}{3}+\frac{(3-3-2)10}{4}=10$$

The branch thus configured is shown in figure 3.

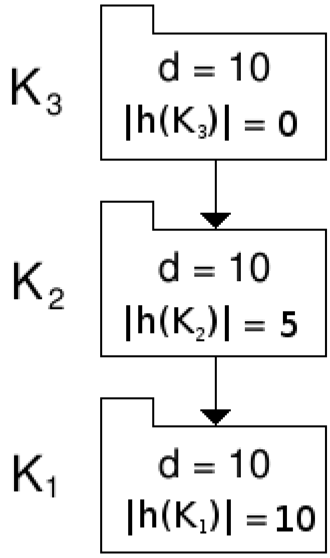

Figure 3: A branch non-uniformly distributed in hidden elements with minimised potential coupling.

Calculating the potential coupling of this minimised branch we see the following.

The potential coupling of K_3 is *10x9+10x20=290.*

The potential coupling of K_2 is *15x14+15x10=360.*

The potential coupling of K_1 is *20x19 = 380.*

The branch's total potential coupling is *290+360+380=1030*, down from the original, uniformly distributed figure of *1080*.

3. Observations

The reduction of potential coupling achievable using the equation above to guide hidden element distribution is a function of the specific violation, *d*, the number of violational elements per disjoint primary set. In a well-encapsulated set, *d* is small, therefore the reduction of potential coupling achievable using this distribution will not be great in well-encapsulated sets of sets.

It is noteworthy, however, that this hidden element distribution favours packing the hidden elements towards the lower end of the branch. In software development, for example, this argues against, "Top heavy," branches and recommends that leaf subsystems and leaf packages should not contain an above average number of program units.

4. Conclusion

When a branch is uniformly distributed in violational elements, its minimum potential coupling is not achieved by also uniformly distributing its hidden elements. Instead, an equation describes the hidden element distribution to achieve this minimum potential coupling and this equation suggests placing more hidden elements towards the lower part of the branch than the higher.

5. Appendix A

5.1 Propositions

Proposition 9.1

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the internal potential coupling of the i^{th} primary set, $|s_{\text{in}}(Q_i)|$, is given by:

$$|s_{in}(Q_i)| = |h(K_i)|^2 + 2d|h(K_i)| - |h(K_i)| - d + d^2$$

Proof:

By definition [D1.2] in [1], given a primary set Q_i the internal potential coupling $|s_{in}(Q_i)|$ is given by:

$$|s_{in}(Q_i)| = |K_i|(|K_i| - 1) \qquad \text{(i)}$$

By definitions [D1.2], [D1.6] and [D1.7] of [1], the elements in K_i are those in the intersection of K_i with *H* and *V*, or:

$$K_i = v(K_i) \cup h(K_i) \qquad \text{(ii)}$$

By definition [D1.1] in [1], *H* and *V* are disjoint so taking the cardinality of (ii) gives:

$$|K_i| = |v(K_i)| + |h(K_i)| \qquad \text{(iii)}$$

By definition, each disjoint primary set has an information-hiding violation of *d*, so substituting into (iii) gives:

$$|K_i| = d + |h(K_i)| \qquad \text{(iv)}$$

Substituting (iv) into (i) gives:

$$|s_{in}(Q_i)| = (d + |h(K_i)|)(d + |h(K_i)| - 1)$$

$$= |h(K_i)|^2 + 2d|h(K_i)| - |h(K_i)| - d + d^2$$

QED

Proposition 9.2

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the external potential coupling of the i^{th} primary set, $|s_{ex}(Q_i)|$, is given by:

$$|s_{ex}(Q_i)| = di|h(K_i)| - d|h(K_i)| + d^2 i - d^2$$

Proof:

By proposition 8.11 in [3], given a primary set Q_i of a single branch the external potential coupling is $|s_{ex}(Q_i)|$ given by:

$$|s_{ex}(Q_i)| = |K_i|((i-1)d) \qquad \text{(i)}$$

By definitions [D1.2], [D1.6] and [D1.7] of [1], the elements in K_i are those in the intersection of K_i with *H* and *V*, or:

$$K_i = v(K_i) \cup h(K_i) \qquad \text{(ii)}$$

By definition [D1.1] in [1], *H* and *V* are disjoint so taking the cardinality of (ii) gives:

$$|K_i| = |v(K_i)| + |h(K_i)| \qquad \text{(iii)}$$

By definition, each disjoint primary set has an information-hiding violation of *d*, so substituting into (iii) gives:

$$|K_i| = d + |h(K_i)| \qquad \text{(iv)}$$

Substituting (iv) into (i) gives:

$$|s_{ex}(Q_i)| = (d + |h(K_i)|)((i-1)d)$$
$$= di|h(K_i)| - d|h(K_i)| + d^2 i - d^2$$

QED

Proposition 9.3

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the potential coupling of the i^{th} primary set, $|s(Q_i)|$, is given by:

$$|s(Q_i)| = |h(K_i)|^2 - |h(K_i)| - d + d|h(K_i)| + di|h(K_i)| + d^2 i$$

Proof:

By proposition 9.1, the internal potential coupling of the i^{th} primary set, $|s_{\text{in}}(Q_i)|$, is given by:

$$|s_{in}(Q_i)| = |h(K_i)|^2 + 2d|h(K_i)| - |h(K_i)| - d + d^2 \qquad \text{(i)}$$

By proposition 9.2, the external potential coupling of the i^{th} primary set, $|s_{ex}(Q_i)|$, is given by:

$$|s_{ex}(Q_i)| = di|h(K_i)| - d|h(K_i)| + d^2 i - d^2 \qquad \text{(ii)}$$

By proposition 8.7 in [3], the potential coupling of Q_i is the sum of the external potential coupling sets of Q_i and the internal potential coupling of Q_i, or:

$$|s(Q_i)|=|s_{ex}(Q_i)|+|s_{in}(Q_i)| \qquad \text{(iii)}$$

Substituting (i) and (ii) into (iii) gives:

$$|s(Q_i)|=|h(K_i)|^2-|h(K_i)|-d+d|h(K_i)|+di|h(K_i)|+d^2 i$$

QED

Proposition 9.4

Given a branch B in encapsulated set G of n elements and of r disjoint primary sets, with each disjoint primary set having an information-hiding violation of d, the potential coupling of B, $|s(B)|$, is given by:

$$|s(B)|=\sum_{i=1}^{r}|h(K_i)|^2-\sum_{i=1}^{r}|h(K_i)|-rd+d\sum_{i=1}^{r}|h(K_i)|+d\sum_{i=1}^{r}i|h(K_i)|+d^2\sum_{i=1}^{r}i$$

Proof:

By definition I SHOULD PROVE THIS ???????????????????????xxx, the potential coupling of a branch is the sum of the potential couplings of all its sets, or:

$$|s(B)|=\sum_{i=1}^{r}|s(Q_i)| \qquad \text{(i)}$$

By proposition 9.3, the potential coupling of the i^{th} primary set, $|s(Q_i)|$, is given by:

$$|s(Q_i)|=|h(K_i)|^2-|h(K_i)|-d+d|h(K_i)|+di|h(K_i)|+d^2 i \qquad \text{(ii)}$$

Substituting (ii) into (i) gives:

$$|s(B)|=\sum_{i=1}^{r}(|h(K_i)|^2-|h(K_i)|-d+d|h(K_i)|+di|h(K_i)|+d^2 i)$$

$$=\sum_{i=1}^{r}|h(K_i)|^2-\sum_{i=1}^{r}|h(K_i)|-rd+d\sum_{i=1}^{r}|h(K_i)|+d\sum_{i=1}^{r}i|h(K_i)|+d^2\sum_{i=1}^{r}i$$

QED

Proposition 9.5

Given a branch B in encapsulated set G of n elements and of r disjoint primary sets, with each disjoint primary set having an information-hiding violation of d, the potential coupling of B, $|s(B)|$, is given by:

$$|s(B)|=|h(K_1)|^2+\sum_{i=2}^{r}|h(K_i)|^2-|h(K_1)|-\sum_{i=2}^{r}|h(K_i)|-rd$$
$$+d|h(K_1)|+d\sum_{i=2}^{r}|h(K_i)|+d|h(K_1)|+d\sum_{i=2}^{r}i|h(K_i)|+d^2\sum_{i=2}^{r}i$$

Proof:

By proposition 9.4, the potential coupling of B, $|s(B)|$, is given by:

$$|s(B)| = \sum_{i=1}^{r} |h(K_i)|^2 - \sum_{i=1}^{r} |h(K_i)| - rd + d\sum_{i=1}^{r} |h(K_i)| + d\sum_{i=1}^{r} i|h(K_i)| + d^2 \sum_{i=1}^{r} i \quad \text{(i)}$$

By definition:

$$\sum_{i=1}^{r} x_i = x_1 + \sum_{i=2}^{r} x_i \quad \text{(ii)}$$

Applying (ii) to (i) gives:

$$|s(B)| = |h(K_1)|^2 + \sum_{i=2}^{r} |h(K_i)|^2 - |h(K_1)| - \sum_{i=2}^{r} |h(K_i)| - rd$$
$$+ d|h(K_1)| + d\sum_{i=2}^{r} |h(K_i)| + d|h(K_1)| + d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i$$

$$= \begin{matrix} |h(K_1)|^2 + \sum_{i=2}^{r} |h(K_i)|^2 - |h(K_1)| - \sum_{i=2}^{r} |h(K_i)| - rd \\ + 2d|h(K_1)| + d\sum_{i=2}^{r} |h(K_i)| + d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i \end{matrix}$$

QED

Proposition 9.6

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the number of elements in *B* is given by:

$$n = \sum_{i=1}^{r} |h(K_i)| + rd$$

Proof:

By definition, the number of elements in a branch is the sum of the number of elements in all its disjoint primary sets, or:

$$|B| = n = \sum_{i=1}^{r} |K_i| \quad \text{(i)}$$

By definitions [D1.2], [D1.6] and [D1.7] of [1], the elements in K_i are those in the intersection of K_i with *H* and *V*, or:

$$K_i = v(K_i) \cup h(K_i) \quad \text{(ii)}$$

By definition [D1.1] in [1], *H* and *V* are disjoint so taking the cardinality of (ii) gives:

$$|K_i| = |v(K_i)| + |h(K_i)| \quad \text{(iii)}$$

By definition, each disjoint primary set has an information-hiding violation of *d*, so substituting into (iii) gives:

$$|K_i| = d + |h(K_i)| \quad \text{(iv)}$$

Substituting (iv) into (i) gives:

$$n = \sum_{i=1}^{r} (d + |h(K_i)|)$$

$$= \sum_{i=1}^{r} d + \sum_{i=1}^{r} |h(K_i)|$$

$$= rd + \sum_{i=1}^{r} |h(K_i)|$$

QED

Proposition 9.7

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the number of information-hidden elements in the first primary set, $|h(K_1)|$, is given by:

$$|h(K_1)| = n - \sum_{i=2}^{r} |h(K_i)| - rd$$

Proof:

By proposition 9.6 the number of elements in *B* is given by:

$$n = \sum_{i=1}^{r} |h(K_i)| + rd$$

$$= |h(K_1)| + \sum_{i=2}^{r} |h(K_i)| + rd$$

$$|h(K_1)| = n - \sum_{i=2}^{r} |h(K_i)| - rd$$

QED

Proposition 9.8

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the square of the number of information-hidden elements in the first primary set, $|h(K_1)|^2$, is given by:

$$|h(K_1)|^2 = n^2 - 2n \sum_{i=2}^{r} |h(K_i)| - 2rdn + \sum_{i=2}^{r} |h(K_i)| \sum_{i=2}^{r} |h(K_i)| + 2rd \sum_{i=2}^{r} |h(K_i)| + r^2 d^2$$

Proof:

By proposition 9.7 the number of information-hidden elements in the first primary set, $|h(K_1)|$, is given by:

$$|h(K_1)| = n - \sum_{i=2}^{r} |h(K_i)| - rd \qquad \text{(i)}$$

Squaring both sides of (i) gives:

$$|h(K_1)|^2 = n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2 d^2$$

QED

Proposition 9.9

Given a branch B in encapsulated set G of n elements and of r disjoint primary sets, with each disjoint primary set having an information-hiding violation of d, the potential coupling of B, $|s(B)|$, is given by:

$$|s(B)| = n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2 d^2 + \sum_{i=2}^{r}|h(K_i)|^2$$
$$-n + 2dn - d\sum_{i=2}^{r}|h(K_i)| - 2rd^2 + d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i$$

Proof:

By proposition 9.5 the potential coupling of B, $|s(B)|$, is given by:

$$|s(B)| = |h(K_1)|^2 + \sum_{i=2}^{r}|h(K_i)|^2 - |h(K_1)| - \sum_{i=2}^{r}|h(K_i)| - rd$$
$$+2d|h(K_1)| + d\sum_{i=2}^{r}|h(K_i)| + d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i \qquad \text{(i)}$$

By proposition 9.7 the number of information-hidden elements in the first primary set, $|h(K_1)|$, is given by:

$$|h(K_1)| = n - \sum_{i=2}^{r}|h(K_i)| - rd \qquad \text{(ii)}$$

By proposition 9.8 the square of the number of information-hidden elements in the first primary set, $|h(K_1)|^2$, is given by:

$$|h(K_1)|^2 = n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2 d^2 \qquad \text{(iii)}$$

Substituting (ii) and (iii) into (i) gives:

$$|s(B)| = n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2 d^2 + \sum_{i=2}^{r}|h(K_i)|^2$$
$$-(n - \sum_{i=2}^{r}|h(K_i)| - rd) - \sum_{i=2}^{r}|h(K_i)| - rd$$
$$+2d(n - \sum_{i=2}^{r}|h(K_i)| - rd) + d\sum_{i=2}^{r}|h(K_i)|$$
$$+d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i$$

$$= \begin{array}{c} n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2d^2 + \sum_{i=2}^{r}|h(K_i)|^2 \\ -n + \sum_{i=2}^{r}|h(K_i)| + rd - \sum_{i=2}^{r}|h(K_i)| - rd \\ +2dn - 2d\sum_{i=2}^{r}|h(K_i)| - 2rd^2 + d\sum_{i=2}^{r}|h(K_i)| \\ +d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i \end{array}$$

$$= \begin{array}{c} n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2d^2 + \sum_{i=2}^{r}|h(K_i)|^2 \\ -n + 2dn - d\sum_{i=2}^{r}|h(K_i)| - 2rd^2 + d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i \end{array}$$

QED

Proposition 9.10

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the number of information-hidden elements in the i^{th} primary set, $|h(K_i)|$, where $i \neq 1$, which minimises the potential coupling of *B* is given by:

$$|h(K_i)| = |h(K_1)| - \frac{(i-1)d}{2}$$

Proof:

By proposition 9.9, the potential coupling of *B*, $|s(B)|$, is given by:

$$\begin{array}{c} |s(B)| = n^2 - 2n\sum_{i=2}^{r}|h(K_i)| - 2rdn + \sum_{i=2}^{r}|h(K_i)|\sum_{i=2}^{r}|h(K_i)| + 2rd\sum_{i=2}^{r}|h(K_i)| + r^2d^2 + \sum_{i=2}^{r}|h(K_i)|^2 \\ -n + 2dn - d\sum_{i=2}^{r}|h(K_i)| - 2rd^2 + d\sum_{i=2}^{r} i|h(K_i)| + d^2\sum_{i=2}^{r} i \end{array}$$

(i)

To find the value of $|h(K_i)|$ which minimises this potential coupling we must differentiate (i) w.r.t. $|h(K_i)|$ and set to zero:

$$\frac{\partial|s(B)|}{\partial|h(K_{i,i\neq 1})|} = -2n + 2\sum_{i=2}^{r}|h(K_i)| + 2rd + 2|h(K_i)| - d + di = 0$$

$$2|h(K_i)| = 2n - 2\sum_{i=2}^{r}|h(K_i)| - 2rd + d - di \qquad \text{(ii)}$$

By proposition 9.7, the number of information-hidden elements in the first primary set, $|h(K_1)|$, is given by:

$$|h(K_1)| = n - \sum_{i=2}^{r} |h(K_i)| - rd$$

$$\sum_{i=2}^{r} |h(K_i)| = n - |h(K_1)| - rd \qquad \text{(iii)}$$

Substituting (iii) into (ii) gives:

$$2|h(K_i)| = 2n - 2(n - |h(K_1)| - rd) - 2rd + d - di$$

$$= \; 2n - 2n + 2|h(K_1)| + 2rd - 2rd + d - di$$

$$|h(K_i)| = |h(K_1)| - \frac{(i-1)d}{2}$$

QED

Proposition 9.11

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the number of information-hidden elements in the first primary set, $|h(K_1)|$ which minimises the potential coupling of *B* is given by:

$$|h(K_1)| = \frac{n}{r} + \frac{(r-5)d}{4}$$

Proof:

By proposition 9.7, the number of information-hidden elements in the first primary set, $|h(K_1)|$, is given by:

$$|h(K_1)| = n - \sum_{i=2}^{r} |h(K_i)| - rd \qquad \text{(i)}$$

By proposition 9.10, the number of information-hidden elements in the i^{th} primary set, $|h(K_i)|$, where $i \neq 1$, which minimises the potential coupling of *B* is given by:

$$|h(K_i)| = |h(K_1)| - \frac{(i-1)d}{2} \qquad \text{(ii)}$$

Substituting (ii) into (i) gives:

$$|h(K_1)| = n - \sum_{i=2}^{r} \left(|h(K_1)| - \frac{(i-1)d}{2}\right) - rd$$

$$= \; n - \sum_{i=2}^{r} |h(K_1)| + \sum_{i=2}^{r} \frac{di}{2} - \sum_{i=2}^{r} \frac{d}{2} - rd$$

$$= \; n - (r-1)|h(K_1)| + \left(\sum_{i=1}^{r} \frac{di}{2} - \frac{d}{2}\right) - (r-1)\frac{d}{2} - rd$$

$$= \quad n - r|h(K_1)| + |h(K_1)| + \sum_{i=1}^{r} \frac{di}{2} - \frac{d}{2} - \frac{rd}{2} + \frac{d}{2} - rd$$

$$= \quad n - r|h(K_1)| + \frac{d}{2}\sum_{i=1}^{r} i - \frac{rd}{2} - rd = 0$$

$$= \quad n - r|h(K_1)| + \frac{d}{2}\frac{r(r+1)}{2} - \frac{3rd}{2}$$

$$r|h(K_1)| = n + \frac{r(r+1)d}{4} - \frac{3rd}{2}$$

$$|h(K_1)| = \frac{n}{r} + \frac{(r+1)d}{4} - \frac{3d}{2}$$

$$= \quad \frac{n}{r} + \frac{rd + d - 6d}{4}$$

$$= \quad \frac{n}{r} + \frac{(r-5)d}{4}$$

QED

Proposition 9.12

Given a branch *B* in encapsulated set *G* of *n* elements and of *r* disjoint primary sets, with each disjoint primary set having an information-hiding violation of *d*, the number of information-hidden elements in the i^{th} primary set, $|h(K_i)|$ which minimises the potential coupling of *B* is given by:

$$|h(K_i)| = \frac{n}{r} + \frac{(r-3-2i)d}{4}$$

Proof:

By proposition 9.10, the number of information-hidden elements in the i^{th} primary set, $|h(K_i)|$, where $i \neq 1$, which minimises the potential coupling of *B* is given by:

$$|h(K_i)| = |h(K_1)| - \frac{(i-1)d}{2} \qquad \text{(i)}$$

By proposition 9.11, the number of information-hidden elements in the first primary set, $|h(K_1)|$ which minimises the potential coupling of *B* is given by:

$$|h(K_1)| = \frac{n}{r} + \frac{(r-5)d}{4} \qquad \text{(ii)}$$

Substituting (ii) into (i) gives:

$$|h(K_i)| = \frac{n}{r} + \frac{(r-5)d}{4} - \frac{(i-1)d}{2}$$

$$= \quad \frac{n}{r} + \frac{(r-5-2\mathrm{i}+2)d}{4}$$

$$= \frac{n}{r} + \frac{(r - 3 - 2\mathrm{i})\,d}{4}$$

QED

6. References

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